# Problem Solving on Vectors

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BY DHIR SIR.....

- A vector perpendicular to $\overset{\vec{A}}{\hat{\imath} + \hat{\jmath} + \hat{k}}$ is

1. $\hat{i} - \hat{j} + \hat{k}$
2. $\hat{i} - \hat{j} - \hat{k}$
3. $-\hat{i} - \hat{j} - \hat{k}$
4. [✓ $3\hat{i} + 2\hat{j} - 5\hat{k}$]

$$\vec{A} \cdot \vec{B} = 0 \Rightarrow A_x B_x + A_y B_y + A_z B_z = 0$$

$$A_x = 1, \; A_y = 1, \; A_z = 1$$

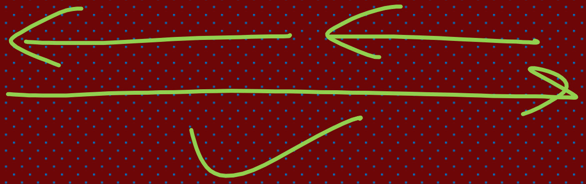
$$① \; \cancel{1 \times 1} + \cancel{1 \times -1} + 1 \times 1 = 1$$

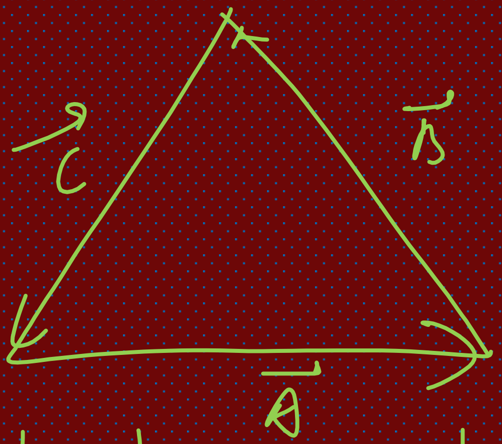
$$④ \; 3 \times 1 + 2 \times 1 - 5 \times 1 = 0$$

- Out of the following set of forces, the resultant of which cannot be zero?

1. 10, 10, 10
2. 10, 10, 20
3. 10, 20, 20
4. 10, 20, 40

$$\vec{A}+\vec{B}+\vec{C}=0$$

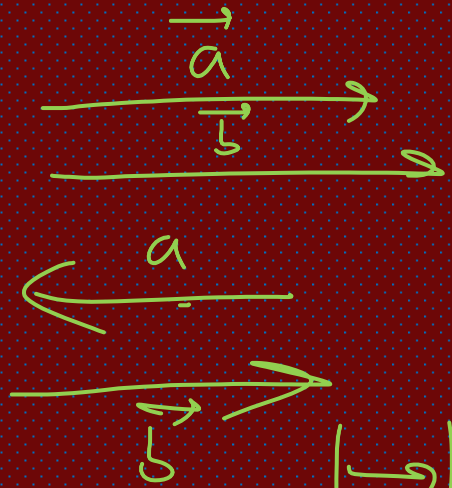




$$|\vec{A}|+|\vec{B}| \geq |\vec{C}|$$

- The ratio of maximum and minimum magnitudes of the resultant of two vectors $\vec{a}$ *and* $\vec{b}$ is 3 : 1. Now $|\vec{a}|$ is equal to

1. $|\vec{b}|$
2. [✓ $2|\vec{b}|$]
3. $3|\vec{b}|$
4. $4|\vec{b}|$



$|\vec{a}| + |\vec{b}| \Rightarrow$ max

$|\vec{a}| - |\vec{b}| \Rightarrow$ Min

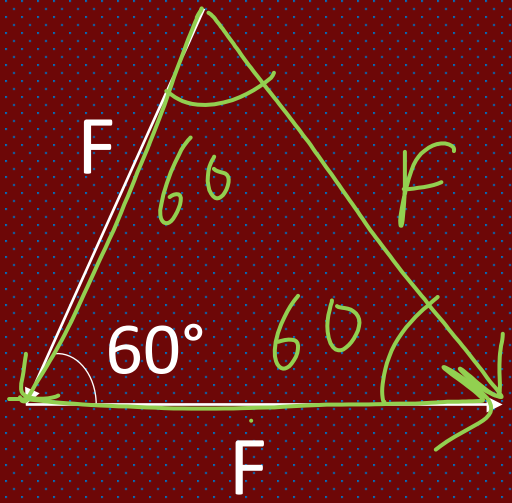
$$\frac{|\vec{a}| + |\vec{b}|}{|\vec{a}| - |\vec{b}|} = 3 \Rightarrow |\vec{a}| + |\vec{b}| = 3|\vec{a}| - 3|\vec{b}|$$

$$2\cancel{4}|\vec{b}| = \cancel{2}|\vec{a}|$$

$$|\vec{a}| = 2|\vec{b}|$$

- Two forces, each equal to F, act as shown in Fig. Their resultant is

1. F/2
2. [✓ F]
3. $\sqrt{3}$ F
4. $\sqrt{5}$ F



- Vector $\vec{A}$ is 2 cm long and is 60° above the x – axis in the first quadrant. Vector $\vec{B}$ is 2 cm long and is 60° below the x – axis in the fourth quadrant. The sum $\vec{A}+\vec{B}$ is a vector of magnitude

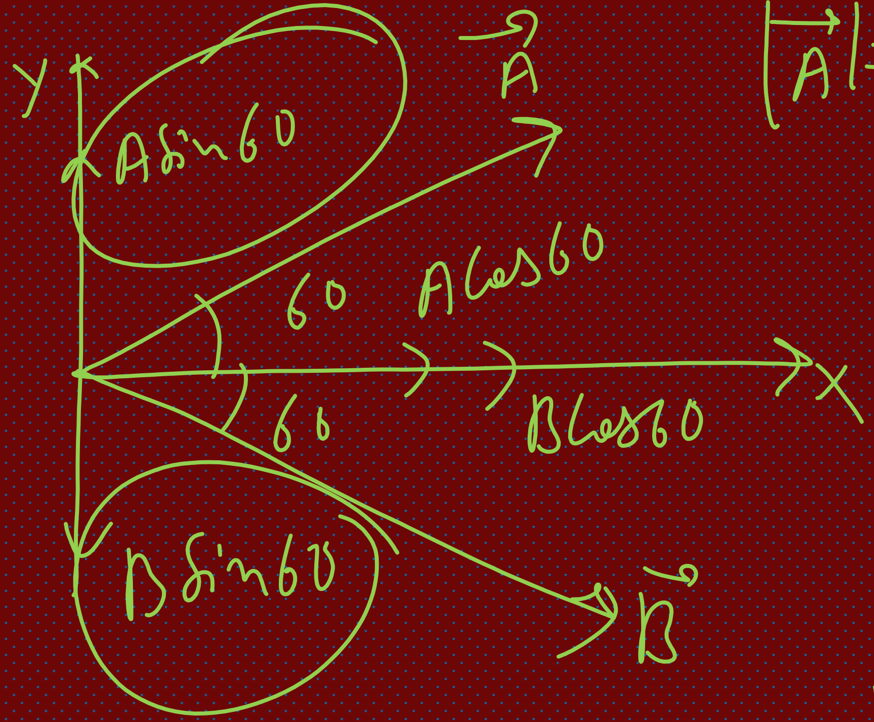
1. 2 cm along positive y – axis

[✓ 2. 2 cm along positive x – axis]

3. 2 cm along negative y – axis

4. 2 cm along negative x – axis

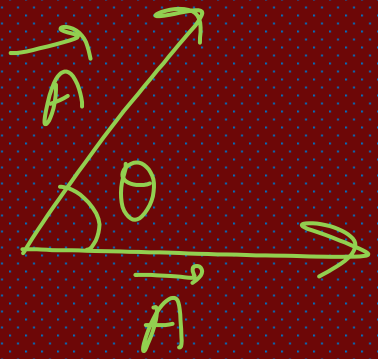
$|\vec{A}| = |\vec{B}| = 2\,cm$



$$\text{Resultant} = A\cos 60 + B\cos 60 = 2\times\frac{1}{2} + 2\times\frac{1}{2}$$
$$= 2\,cm$$

- What is the angle between two vector forces of equal magnitude such that their resultant is one – third of either of the original forces?

1. ✓ $\cos^{-1}\left(-\frac{17}{18}\right)$
2. $\cos^{-1}\left(-\frac{1}{3}\right)$
3. 45°
4. 120°



$$|\vec{R}| = \frac{|\vec{A}|}{3}$$

$$|\vec{R}| = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

$$\frac{A}{3} = \sqrt{A^2 + A^2 + 2A^2\cos\theta}$$

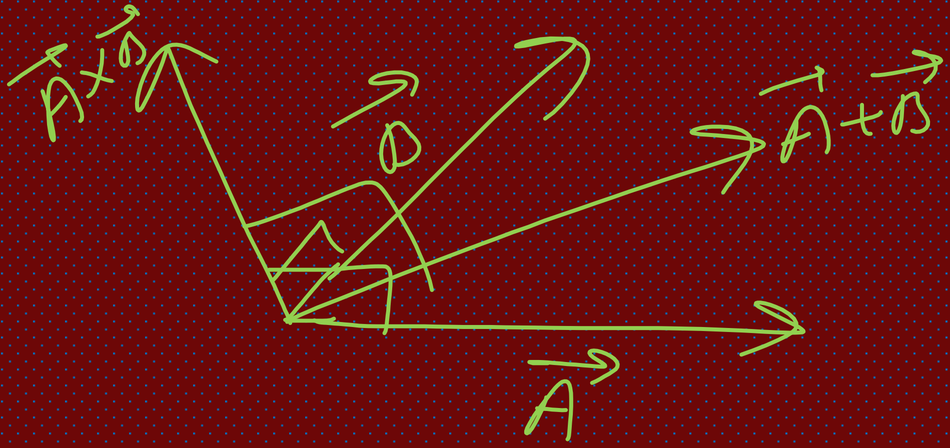
$$\frac{\cancel{A^2}}{9} = 2\cancel{A^2}(1+\cos\theta)$$

$$1+\cos\theta = \frac{1}{18}$$

$$\cos\theta = \frac{1}{18} - 1 = -\frac{17}{18}$$

$$\boxed{\theta = \cos^{-1}\left(-\frac{17}{18}\right)}$$

- The angle between $\vec{A}+\vec{B}$ and $\vec{A}\times\vec{B}$ is



1. 0
2. π/4
3. [✓ π/2]
4. π

- The projection of a vector $\vec{r} = 3\hat{\imath} + \hat{\jmath} + \hat{k}$ on the x – y plane has magnitude

1. 3
2. 4
3. $\sqrt{14}$
4. [✓ $\sqrt{10}$]

$$\sqrt{3^2 + 1^2}$$

$$= \sqrt{10}$$

- If $|\vec{A}+\vec{B}| = |\vec{A}| = |\vec{B}|$, then the angle between $\vec{A}$ and $\vec{B}$ is

1. [✓ 120°]
2. 60°
3. 90°
4. 0°

$$|\vec{A}+\vec{B}| = \sqrt{A^2+B^2+2AB\cos\theta}$$

$$A = \sqrt{A^2+A^2+2A^2\cos\theta}$$

$$\cancel{A^2} = 2\cancel{A^2}(1+\cos\theta)$$

$$\cos\theta = \frac{1}{2}-1 = -\frac{1}{2} \Rightarrow \theta = 120$$

- If vector $\vec{A} = \hat{\imath} + 2\hat{\jmath} + 4\hat{k}$ and $\vec{B} = 5\hat{\imath}$ represent the two sides of a triangle, then the third side of the triangle can have length equal to

$$|\vec{A}| = \sqrt{1+2^2+4^2} = \sqrt{21} \simeq 4\,6$$

$$|\vec{B}| = 5 \,, \quad \sqrt{56} \simeq 7.5$$

1. 6
2. $\sqrt{56}$
3. [✓ Both of the above]
4. None of the above

$\underline{4\,6,\ 5,\ 7\,5}$ △ ✓ | $4\,6,\ 5,\ 6$ △ ✓

- Three vectors $\vec{A}, \vec{B}, \vec{C}$ satisfy the relation $\vec{A}.\vec{B} = 0$ and $\vec{A}.\vec{C} = 0$. The vector $\vec{A}$ is parallel to

1. $\vec{B}$
2. $\vec{C}$
3. $\vec{B}.\vec{C}$
4. [✓ $\vec{B} \times \vec{C}$]

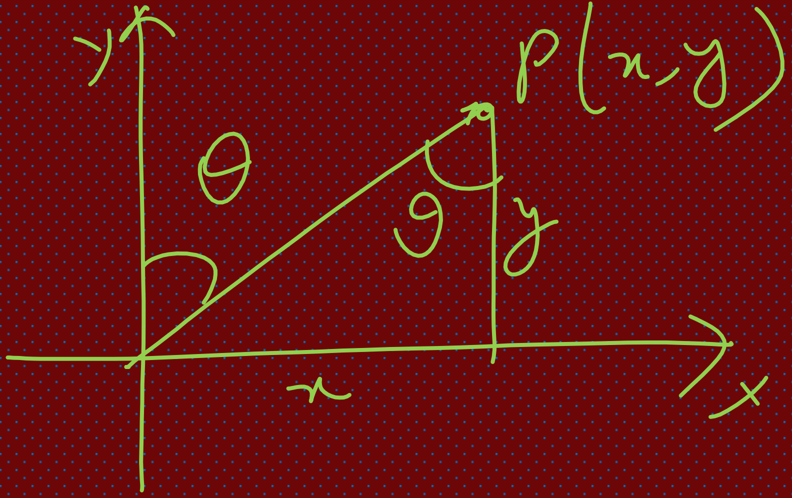
$\vec{A}.\vec{B} = 0 \Rightarrow \theta_1 = 90$

$\vec{A}.\vec{C} = 0 \Rightarrow \theta_2 = 90$

$\vec{B} \times \vec{C} \perp \vec{B}$ and $\vec{C}$

- The angle which the vector $\vec{A} = 2\hat{\imath} + 3\hat{\jmath}$ makes with the y – axis where $\hat{\imath}$ and $\hat{\jmath}$ are unit vectors along x – and y – axes respectively, is

1. $\cos^{-1}\left(\frac{3}{5}\right)$
2. $\cos^{-1}\left(\frac{2}{3}\right)$
3. [✓ $\tan^{-1}\left(\frac{2}{3}\right)$]
4. $\sin^{-1}\left(\frac{2}{3}\right)$



$$\tan\theta = \frac{x}{y}$$

$$\tan\theta = \frac{2}{3}$$

$$\boxed{\theta = \tan^{-1}\left(\frac{2}{3}\right)}$$

- Two vectors $\vec{a}\ and\ \vec{b}$ are such that $|\vec{a}+\vec{b}| = |\vec{a}-\vec{b}|$. What is the angle between $\vec{a}$ and $\vec{b}$ ?

1. 0°
2. [✓ 90°]
3. 60°
4. 180°

$$|\vec{a}+\vec{b}|^2 = |\vec{a}-\vec{b}|^2$$

$$\cancel{a^2}+\cancel{b^2}+2ab\cos\theta = \cancel{a^2}+\cancel{b^2}-2ab\cos\theta$$

$$4ab\cos\theta = 0 \Rightarrow \cos\theta = 0$$

$$\theta = 90$$

- Given $\vec{A} = 2\hat{\imath} + p\hat{\jmath} + q\hat{k}\ and\ \vec{B} = 5\hat{\imath} + 7\hat{\jmath} + 3\hat{k}$. If $\vec{A}\,||\vec{B}$, then the values of p and q are, respectively,

[✓ 1. $\frac{14}{5}$ and $\frac{6}{5}$]

2. $\frac{14}{3}$ and $\frac{6}{5}$

3. $\frac{6}{5}$ and $\frac{1}{3}$

4. $\frac{3}{4}$ and $\frac{1}{4}$

$$\vec{A}\,||\,\vec{B}$$

$$\frac{A_x}{B_x} = \frac{A_y}{B_y} = \frac{A_z}{B_z}$$

$$\frac{2}{5} = \frac{p}{7} = \frac{q}{3}$$

$$p = \frac{14}{5}$$

$$q = \frac{6}{5}$$

- If $\vec{A}$ is perpendicular to $\vec{B}$, then

1. $\vec{A} \times \vec{B} = 0$ ✗
2. $\vec{A}.\vec{B} = AB$ ✗
3. ✓ $\vec{A}.[\vec{A} + \vec{B}] = A^2$
4. $\vec{A}.[\vec{A} + \vec{B}] = A^2 + AB$

$$\vec{A}\,\vec{B} = 0$$

$$\vec{A}\cdot(\vec{A} + \vec{B}) = \vec{A}\cdot\vec{A} + \vec{A}\cdot\vec{B}$$

$$= A^2 + 0$$

$$= A^2$$